**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans) x = 60- 10 🡺 50; z = x-µ/🡺 50-45/8= 5/8🡺0.625

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.   
   Ans) z = x-µ/🡺 44-38/6 = 5/6🡺1 zscore = 0.84134=84.13%, z = x-µ/🡺 38-38/6 = 0🡺0 zscore = 50% , between 38 and 44 🡺 zscore 38 - zscore 44🡺34.13  
   34.13 % in 400 , 136 in 400 is not even near to its middle value so employees at the processing center are older than 44 than between 38 and 44 will be false
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.  
   Ans) z = x-µ/🡺 30-38/6🡺 -8/6= -1.3 zscore is 0.09126 🡺9.15% in 400 is 36 in 400 so for employees under the age of 30 at the center would be expected to attract about 36 employees is True
4. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans) 2x1 will also be normal distribution and x1 + x2 will also be similar to normal distribution so 2 *X*1 and *X*1 + *X*2  there will be no major difference between them .

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) X ~ N(100, 202). Then µ= 100 = 20, so p (a<x<b) from the above details, we have to exclude area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Z score of .005 = stats.norm.ppf(0.005)🡺 -2.575

Z score of .995 = stats.norm.ppf(0.995)🡺 2.575

Z= x-100/20 🡺 x= 20(z) + 100

a= -(20 \* 2.575 + 100 ); b= -(20 \* 2.575 + 100)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans)

import numpy as np

from scipy import stats

from scipy.stats import norm

# Mean profits from two different divisions of a company = Mean1 + Mean2

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million') 🡪 Mean Profit is Rs 540 Million

*# Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2*

SD **=** np**.**sqrt((9)**+**(16))

print('Standard Deviation is Rs', SD**\***45, 'Million')🡪 Standard Deviation is Rs 225.0 Million

*# A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.*

print('Range is Rs',(stats**.**norm**.**interval(0.95,540,225)),'in Millions')🡪 Range is Rs (99.00810347848784, 980.9918965215122) in Millions

*# B. Specify the 5th percentile of profit (in Rupees) for the company*

*# To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645*

X**=** 540**+**(**-**1.645)**\***(225)

print('5th percentile of profit (in Million Rupees) is',np**.**round(X,))

🡪5th percentile of profit (in Million Rupees) is 170.0

*# C. Which of the two divisions has a larger probability of making a loss in a given year?*

*# Probability of Division 1 making a loss P(X<0)*

stats**.**norm**.**cdf(0,5,3)

0.0477903522728147

*# Probability of Division 2 making a loss P(X<0)*

stats**.**norm**.**cdf(0,7,4)

0.040059156863817086

1. Specify the 5th percentile of profit (in Rupees) for the company
2. Which of the two divisions has a larger probability of making a loss in a given year?